M07/5/MATME/SP2/ENG/TZ1/XX

22077304

MATHEMATICS
STANDARD LEVEL
PAPER 2

Tuesday 8 May 2007 (morning)

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

2207-7304

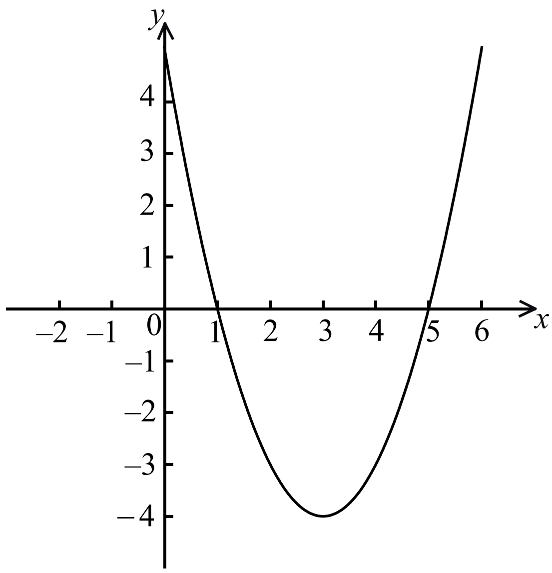
8 pages

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Total mark: 25]*

Part A *[Maximum mark: 10]*

The following diagram shows part of the graph of a quadratic function, with equation in the form $y=(x-p)(x-q)$, where $p, q \in \mathbb{Z}$.



(a) Write down

(i) the value of p and of q;

(ii) the equation of the axis of symmetry of the curve. *[3 marks]*

(b) Find the equation of the function in the form $y=(x-h)^2+k$, where $h, k \in \mathbb{Z}$. *[3 marks]*

(c) Find $\frac{dy}{dx}$. *[2 marks]*

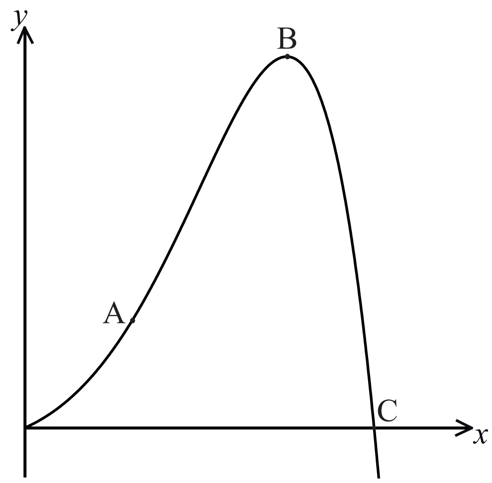
(d) Let T be the tangent to the curve at the point $(0, 5)$. Find the equation of T. *[2 marks]*

(This question continues on the following page)

(Question 1 continued)

Part B *[Maximum mark: 15]*

The function f is defined as $f(x) = \mathrm{e}^x \sin x$, where x is in radians. Part of the curve of f is shown below.



There is a point of inflexion at A, and a local maximum point at B. The curve of f intersects the x-axis at the point C.

(a) Write down the x-coordinate of the point C. *[1 mark]*

(b) (i) Find $f'(x)$.

(ii) Write down the value of $f'(x)$ at the point B. *[4 marks]*

(c) Show that $f''(x) = 2\mathrm{e}^x \cos x$. *[2 marks]*

(d) (i) Write down the value of $f''(x)$ at A, the point of inflexion.

(ii) Hence, calculate the coordinates of A. *[4 marks]*

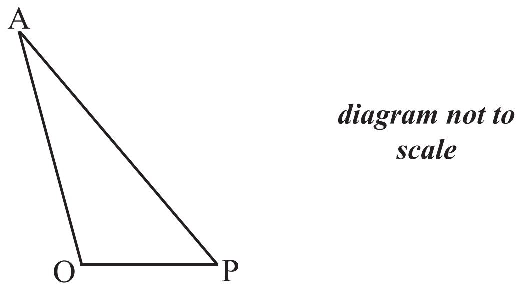
(e) Let R be the region enclosed by the curve and the x-axis, between the origin and C.

(i) Write down an expression for the area of R.

(ii) Find the area of R. *[4 marks]*

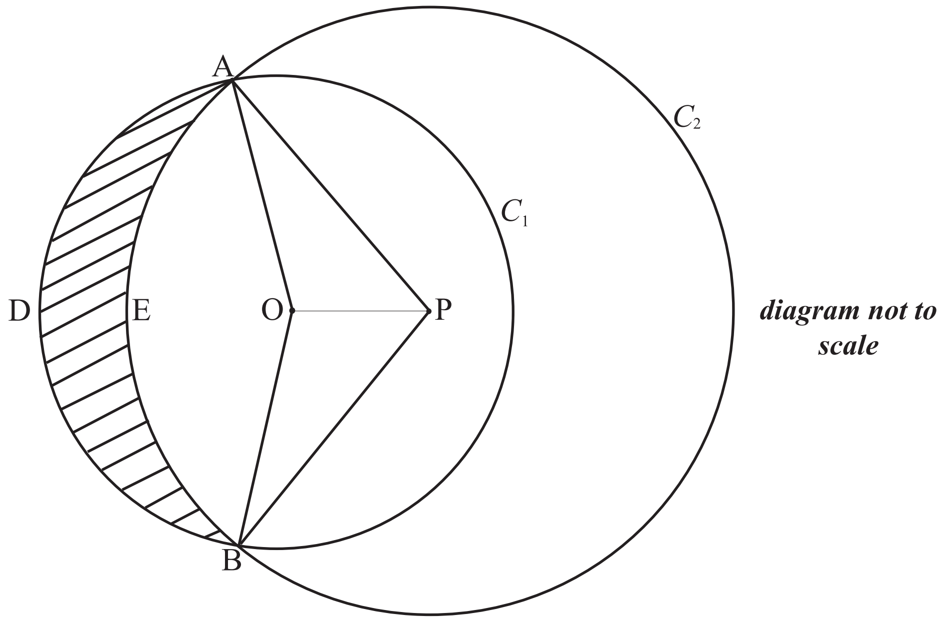
2. *[Maximum mark: 14]*

The following diagram shows the triangle AOP, where $\text{OP} = 2\text{ cm}$, $\text{AP} = 4\text{ cm}$ and $\text{AO} = 3\text{ cm}$.



(a) Calculate $\text{A}\hat{\text{O}}\text{P}$, giving your answer in radians. *[3 marks]*

The following diagram shows two circles which intersect at the points A and B. The smaller circle C_1 has centre O and radius 3 cm, the larger circle C_2 has centre P and radius 4 cm, and $\text{OP} = 2\text{ cm}$. The point D lies on the circumference of C_1 and E on the circumference of C_2. Triangle AOP is the same as triangle AOP in the diagram above.



(b) Find $\text{A}\hat{\text{O}}\text{B}$, giving your answer in radians. *[2 marks]*

(This question continues on the following page)

(Question 2 continued)

(c) Given that $\text{A}\hat{\text{P}}\text{B}$ is 1.63 **radians**, calculate the area of

(i) sector PAEB ;

(ii) sector OADB . *[5 marks]*

(d) The area of the quadrilateral AOBP is $5.81\ \text{cm}^2$.

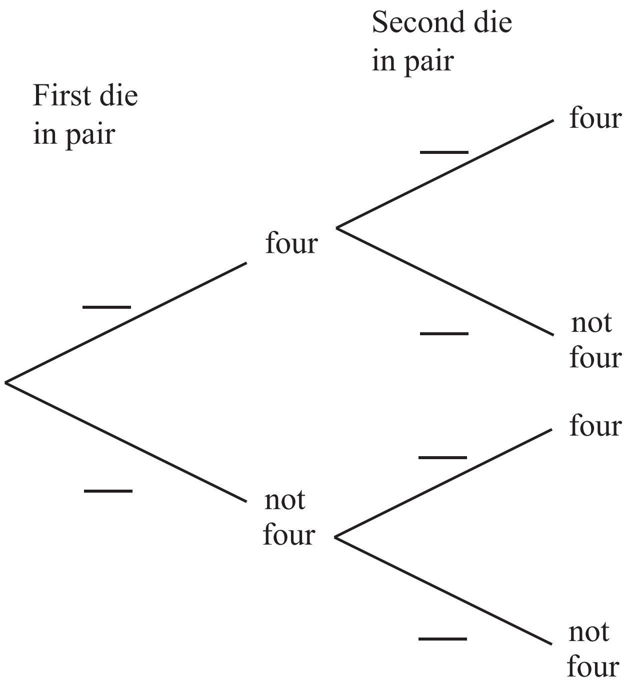
(i) Find the area of AOBE .

(ii) Hence find the area of the shaded region AEBD . *[4 marks]*

3. *[Maximum mark: 12]*

A pair of fair dice is thrown.

(a) Copy and complete the tree diagram below, which shows the possible outcomes.



[3 marks]

Let E be the event that **exactly** one four occurs when the pair of dice is thrown.

(b) Calculate $\mathrm{P}(E)$. *[3 marks]*

The pair of dice is now thrown five times.

(c) Calculate the probability that event E occurs **exactly** three times in the five throws. *[3 marks]*

(d) Calculate the probability that event E occurs **at least** three times in the five throws. *[3 marks]*

4. *[Maximum mark: 22]*

Points P and Q have position vectors $-5\boldsymbol{i}+11\boldsymbol{j}-8\boldsymbol{k}$ and $-4\boldsymbol{i}+9\boldsymbol{j}-5\boldsymbol{k}$ respectively, and both lie on a line L_1.

(a) (i) Find $\overrightarrow{\text{PQ}}$.

(ii) Hence show that the equation of L_1 can be written as

$$\boldsymbol{r}=(-5+s)\boldsymbol{i}+(11-2s)\boldsymbol{j}+(-8+3s)\boldsymbol{k}.$$ *[4 marks]*

The point $\text{R}(2, y_1, z_1)$ also lies on L_1.

(b) Find the value of y_1 and of z_1. *[4 marks]*

The line L_2 has equation $\boldsymbol{r}=2\boldsymbol{i}+9\boldsymbol{j}+13\boldsymbol{k}+t(\boldsymbol{i}+2\boldsymbol{j}+3\boldsymbol{k})$.

(c) The lines L_1 and L_2 intersect at a point T. Find the position vector of T. *[7 marks]*

(d) Calculate the angle between the lines L_1 and L_2. *[7 marks]*

5. *[Maximum mark: 17]*

The function $f(x)$ is defined as $f(x)=3+\frac{1}{2x-5}$, $x\neq\frac{5}{2}$.

(a) Sketch the curve of f for $-5\leq x\leq 5$, showing the asymptotes. *[3 marks]*

(b) Using your sketch, write down

(i) the equation of each asymptote;

(ii) the value of the x-intercept;

(iii) the value of the y-intercept. *[4 marks]*

(c) The region enclosed by the curve of f, the x-axis, and the lines $x=3$ and $x=a$, is revolved through 360° about the x-axis. Let V be the volume of the solid formed.

(i) Find $\int\left(9+\frac{6}{2x-5}+\frac{1}{(2x-5)^2}\right)\mathrm{d}x$.

(ii) Hence, given that $V=\pi\left(\frac{28}{3}+3\ln 3\right)$, find the value of a. *[10 marks]*
